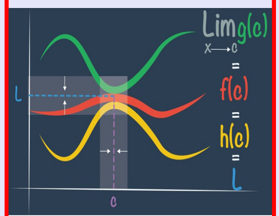
Calculus I

Lecture 28



Feb 19-8:47 AM

Class QZ 27

Given $f(x) = \int_1^{\sqrt{x}} \cos t^2 \, dt$

1) Find $f(1) = \int_1^{\sqrt{1}} \cos t^2 \, dt = \boxed{0}$

2) Find $f'(x)$

$= \cos(\sqrt{x})^2 \cdot \frac{1}{2\sqrt{x}} - \cos 1^2 \cdot 0$

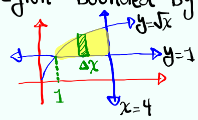
$= \boxed{\frac{\cos x}{2\sqrt{x}}}$

3) Find $f'(1)$ in exact value.

$f'(1) = \frac{\cos 1}{2\sqrt{1}} = \boxed{\frac{\cos 1}{2}}$

Jun 2-8:02 AM

1) Draw the region bounded by $y=\sqrt{x}$, $y=1$, and $x=4$.



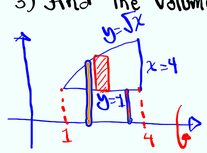
$\sqrt{x}=1$
$x=1$

2) Find its area. $A=\int_1^4[\sqrt{x}-1]dx=\left(\frac{x^{3/2}}{3/2}-x\right)\Big|_1^4$

$=\left(\frac{2x\sqrt{x}}{3}-\frac{3}{3}x\right)\Big|_1^4=\frac{1}{3}\left[2x\sqrt{x}-3x\right]\Big|_1^4=\frac{1}{3}\left[2\cdot4\sqrt{4}-3\cdot4-2\cdot1\sqrt{1}+3\cdot1\right]$

$=\frac{1}{3}\left[16-12-2+3\right]=\boxed{\frac{5}{3}}$

3) Find the volume if we rotate it by x-axis



Washer Method

$R=\sqrt{x}$ $r=1$

$V=\int_1^4\pi\left[R^2-r^2\right]dx$

$=\int_1^4\pi\left[x-1\right]dx=\pi\left[\frac{x^2}{2}-x\right]\Big|_1^4$

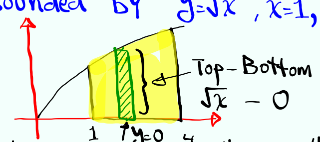
$=\pi\left[8-4-\frac{1}{2}+1\right]$

$=\pi\left[5-\frac{1}{2}\right]=\pi\,4.5$

$=\boxed{4.5\pi}=\boxed{\frac{9\pi}{2}}$

Jun 2-9:07 AM

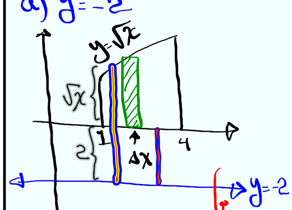
1) Draw the region bounded by $y=\sqrt{x}$, $x=1$, $x=4$, and $y=0$.



2) Find its area. $A=\int_1^4[\sqrt{x}-0]dx=\frac{x^{3/2}}{3/2}\Big|_1^4=\frac{2}{3}x\sqrt{x}\Big|_1^4=\square$

3) Set-up the integral to find the volume if rotate the region by

a) $y=-2$

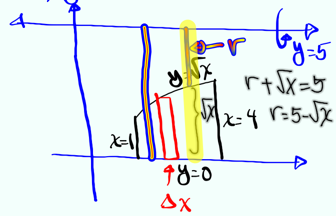


washer

$R=2+\sqrt{x}$ $r=2$

$V=\int_1^4\pi\left[(2+\sqrt{x})^2-(2)^2\right]dx$

b) $y=5$.



washer

$R=5$ $r=5-\sqrt{x}$

$V=\int_1^4\pi\left[5^2-(5-\sqrt{x})^2\right]dx$

Jun 2-9:20 AM

what do we do when ref. rectangle is Parallel to the axis of revolution?

Use cylindrical shell Method

$$V = \int_a^b 2\pi D H \, dx$$

Height of Ref. Rect.

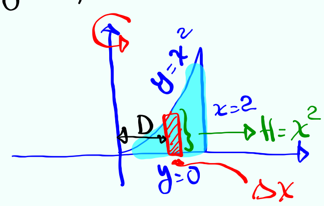
Distance from A.O.R.

$$V = \int_a^b 2\pi D W \, dy$$

width of Ref. Rect.

Jun 2-9:35 AM

Rotate the region bounded by $y=x^2$, $y=0$, and $x=2$ by the y-axis.



$D = x$

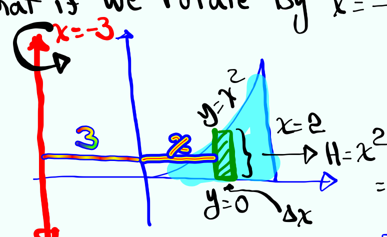
Shell

$D = x$ $H = x^2$

$$V = \int_0^2 2\pi D H \, dx = \int_0^2 2\pi \cdot x \cdot x^2 \, dx$$

$$= 2\pi \cdot \frac{x^4}{4} \Big|_0^2 = \frac{\pi}{2}\left[2^4 - 0^4\right] = \boxed{8\pi}$$

what if we rotate by $x = -3$.



$D = x + 3$

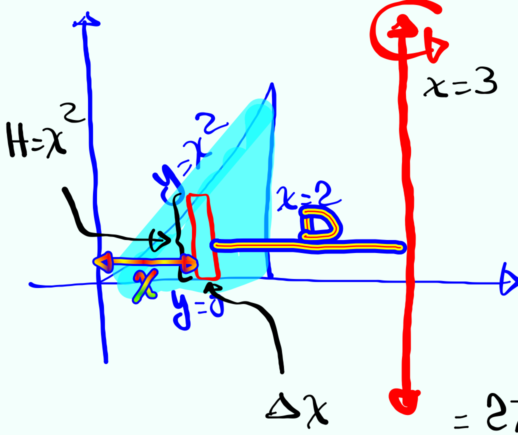
$$V = \int_0^2 2\pi(x+3) \cdot x^2 \, dx$$

$$= 2\pi\left[\frac{x^4}{4} + x^3\right]\Big|_0^2$$

$$= 2\pi\left[4 + 8 - 0\right] = \boxed{24\pi}$$

Jun 2-9:39 AM

Let's rotate by $x = 3$.



$$x + D = 3$$

$$D = 3 - x$$

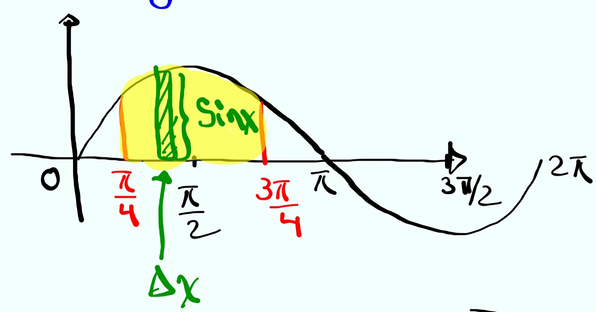
$$V = \int_0^2 2\pi \underbrace{(3-x)}_{D}\,\underbrace{x^2}_{H}\,\underbrace{dx}_{\Delta x}$$

$$= 2\pi \int_0^2 (3x^2 - x^3)\,dx$$

$$= 2\pi \left[x^3 - \frac{x^4}{4} \right]\Bigg|_0^2 = 2\pi[8-4] = \boxed{8\pi}$$

Jun 2-9:51 AM

1) Draw the region bounded by $y = \sin x$, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, and $y = 0$.



2) Find its area.

$$A = \int_{\pi/4}^{3\pi/4} \sin x\,dx = -\cos x \Big|_{\pi/4}^{3\pi/4} = -\left[\cos\frac{3\pi}{4} - \cos\frac{\pi}{4}\right]$$

$$= -\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] = \boxed{\sqrt{2}}$$

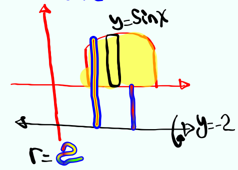
Jun 2-9:57 AM

3) Set-up the integral only to find the Volume if we rotate the enclosed region by

a) $y=-2$

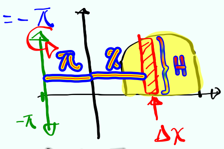
washer

$R = 2 + \sin x$



$$V = \int_{\pi/4}^{3\pi/4} \pi \left[(2+\sin x)^2 - 2^2 \right] dx$$

c) $x = -\pi$



$D = x + \pi$

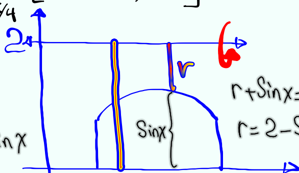
$H = \sin x$

$$V = \int_{\pi/4}^{3\pi/4} 2\pi (x+\pi) \cdot \sin x \, dx$$

b) $y = 2$

$R = 2$

$r = 2 - \sin x$

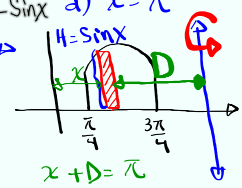


$r + \sin x = 2$

$r = 2 - \sin x$

$$V = \int_{\pi/4}^{3\pi/4} \pi \left[2^2 - (2 - \sin x)^2 \right] dx$$

d) $x = \pi$



$x + D = \pi$

$D = \pi - x$

$$V = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\pi(\pi - x) \cdot \sin x \, dx$$

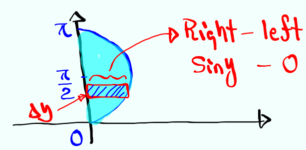
Jun 2-10:05 AM

1) Draw the region bounded by y-axis and $x = \sin y$ in QI.

$y = 0 \to x = 0$

$y = \frac{\pi}{2} \to x = 1$

$y = \pi \to x = 0$



2) Find its area. $A = \int_0^{\pi} \sin y \, dy = -\cos y \Big|_0^{\pi} = \boxed{2}$

3) Rotate it by Y-axis, find the volume.

Disk

$$V = \int_0^{\pi} \pi [\sin y]^2 \, dy = \pi \int_0^{\pi} \sin^2 y \, dy$$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A = \frac{1 - \cos 2A}{2}$

$$= \pi \int_0^{\pi} \frac{1 - \cos 2y}{2} \, dy$$

$$= \frac{\pi}{2} \left[\int_0^{\pi} (1 - \cos 2y) \, dy \right]$$

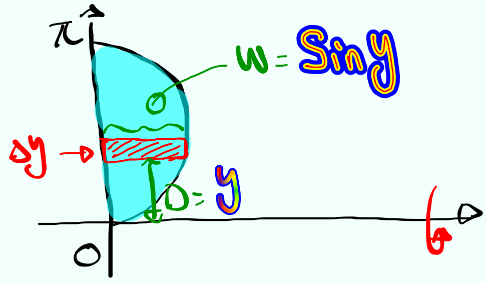
$$= \frac{\pi}{2} \left[\left(y - \frac{1}{2} \sin 2y \right) \Big|_0^{\pi} \right]$$

$$= \frac{\pi}{2} [\pi - 0] = \boxed{\frac{\pi^2}{2}}$$

Jun 2-10:24 AM

3) Rotate by x-axis, Set-up the integral for the volume.

Shell



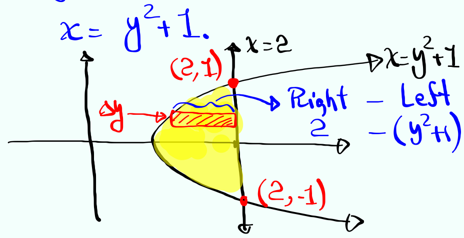
$$V = \int_0^{\pi} 2\pi \cdot y \cdot \sin y \, dy$$

$$= 2\pi \int_0^{\pi} y \sin y \, dy$$

Integration by Parts Calc. II.

Jun 2-10:38 AM

1) Draw the region bounded by $x = 2$ and $x = y^2 + 1$.



2) Set-up the integral for the area.

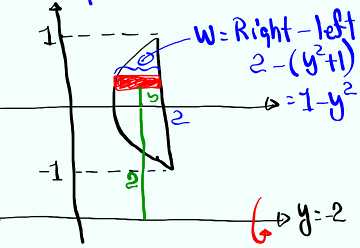
$$A = \int_{-1}^{1} [2 - (y^2+1)] \, dy$$

3) Rotate by $y = -2$, Set-up the integral for the Volume.

Shell

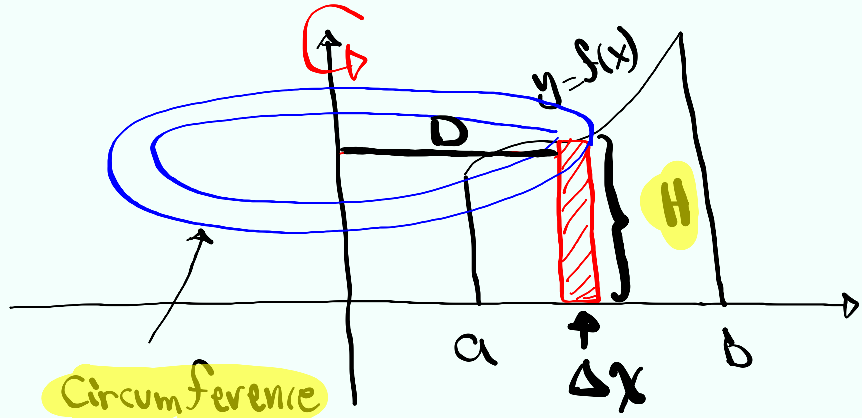
$D = y + 2$ $\quad w = 1 - y^2$

$$V = \int_{-1}^{1} 2\pi (y+2)(1-y^2) \, dy$$



Jun 2-10:43 AM

Rotate the shaded region below by y-axis



$2\pi R = 2\pi D$

$$V = \int_a^b 2\pi D H \, dx$$

Jun 2-10:56 AM

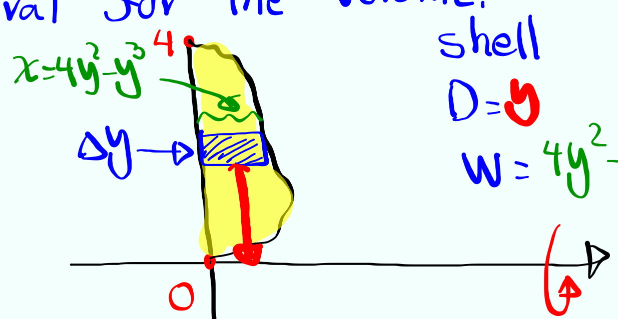
Rotate the region bounded by $x = 4y^2 - y^3$ and $x = 0$ by x-axis. Set-up the integral for the volume.

$x = 0$

$4y^2 - y^3 = 0$

$y^2(4 - y) = 0$

$y = 0 \quad y = 4$



shell

$D = y$

$W = 4y^2 - y^3$

$$V = \int_0^4 2\pi (y)(4y^2 - y^3) \, dy$$

Jun 2-11:02 AM